Do Manipulatives Foster Pre-service Teachers' Understanding of Probability?

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Abstract

Manipulatives have been found to be effective in increasing mathematics achievement when they are put to good use (Canny, 1984; Clements & Battista, 1990; Clements, 1999; Sowell, 1989; Suydam, 1984), but their use in mathematics instruction is not clear and it is often debated to be effective (Ball, 1992; McNeil & Jarvin, 2007). The present study was an attempt to investigate efficacy of manipulatives in facilitating understanding of probability concepts in pre-service teachers. Twenty-five pre-service teachers, enrolled in a probability class, received intervention using manipulatives to promote learning of two key concepts—conceptualizing probability, and understanding theoretical and experimental probability. A pretest-posttest design was employed, and the results revealed efficacy of manipulatives, used as a tool, in promoting learning of the probability concepts. The participants reported that the probability lessons using manipulatives assisted in academic and emotional progress by being informative and useful in addition to an enjoyable experience. The features that may have contributed towards the success and educational implications of the findings are discussed.

Keywords: Manipulatives, Mathematics instruction, Pre-service teachers, Probability

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With mathematics instruction, conceptual understanding can be enhanced by using manipulatives which are the objects designed for a learner to perceive some mathematical concept by manipulating them. That is, manipulatives are concrete objects that students can use as learning tool to model mathematical situations, and that students can act upon in ways that mirror or give meaning to operations. Manipulatives facilitate in understanding by linking concrete model with the symbolic representation, and thus, promote in abstract thinking. The use of manipulatives is specifically helpful for the students in the pre-operational stage when their abstract thinking is developing since they allow them to move from concrete experiences to abstract reasoning (Heddens, 1986; Reisman, 1982; Ross & Kurtz, 1993).The National Council of Teachers of Mathematics (NCTM) *Principles to Action* has recognized manipulatives as an important tool in meaningful understanding of mathematics, and recommended using manipulatives to teach mathematical concepts with students of any age (NCTM, 2014). Research has shown the importance of manipulatives in alleviating behavioral issues among pre-service teachers (Gresham, 2012; Lake & Kelly, 2014; Vinson, 2001), but none of the available research has reported on the efficacy of manipulatives in promoting conceptual understanding of probability. The aim of the present research was to explore the following key, but yet unexamined, pedagogical issue: Do manipulatives help pre-service teachers (PSTs) in understanding probability concepts?

Conceptual Understanding and Role of the Teachers

Recent developments (CCSSO, 2010; NCTM, 2014) emphasizing the need for conceptual knowledge among students set new and ambitious goals for math students that have called for improving the quality of teaching. Improving the quality of education heavily relies on the role of teacher educators and teacher training programs to prepare educators who are ready to succeed in the classrooms (Darling-Hammond, 2012; Wenglinsky,2002), that is, making sure that PSTs receive appropriate training. Teachers need deep content and pedagogical knowledge to support students in learning these goals (Hill, Ball, & Schilling, 2008; Loewenberg, Ball, Thames, & Phelps, 2008). An effective teacher training program involves building content matter knowledge among the teachers along with facilitating in the development of pedagogically sound teaching skills that enable content knowledge available to students in the classroom. Teachers with in-depth mathematics training have been found to effectively connect pedagogical content knowledge and content knowledge (Krauss, Baumert, & Blum, 2008) and creating a mathematics instruction environment that reflect on students' representation of mathematical ideas (Clements, 1999). This calls for teaching PSTs in ways that support them in developing this deep knowledge, and in ways that model the kind of pedagogies they are expected to practice. One area of content taught in the schools is probability and manipulatives present one of the pedagogical tools that aid in enhancing students' content knowledge by providing visual representation to model a problem.

Using Manipulatives as a Pedagogical Tool

Researchers have supported the use of manipulatives as a long-range pedagogical tool to discover a relational concept, or highlight and practice steps in a formal concept (Boggan, Harper, & Whitmire, 2010; Clements & Sarama, 2012). However, some researchers have debated their effectiveness in mathematics instruction (Ball, 1992; McNeil & Jarvin, 2007; Puchner, L., Taylor, A., O'Donnell, B., & Fick, K., 2008) and reported that manipulatives cannot be used as a tool guaranteed to be successful in promoting understanding of a concept (Ball, 1992; Baroody, 1989; Clements & McMillan, 1996). The rationale for using manipulatives actively engages students in hands-on experiences that provide insight into the process of actual computation and thus, leading to conceptual understanding. Both virtual and physical manipulatives were found to be more effective than the traditional instruction (Baki, Kosa, & Guven, 2011) and their use facilitated students in visualizing mathematical relationships (Roschelle et al., 2010). Manipulatives were linked to increasing arithmetic content knowledge, reducing arithmetic misconception, improving students' performance and honing students' mathematical skills (Green, Piel, & Flowers, 2008; Reimer & Moyer, 2005; Stein and Bovalino, 2001).

Manipulatives with PSTs

The efficacy of manipulatives among PSTs is a critical issue as teachers play a pivotal role in enhancing student learning. Teachers are required to enhance students' ability to grapple with problems and to develop a deeper understanding of the mathematics involved. This makes teachers' positive dispositions along with proper strategies to use the manipulatives an important step in influencing student learning. Statistical Education of Teachers (Franklin et al., 2015) emphasized the importance of manipulatives (such as cubes to represent individual data points) in teacher preparation and noted that teachers must be proficient in using manipulatives to aide in the collection, exploration and analysis, and interpretation of data. Research has linked the use of manipulatives in a mathematics instruction with positive behavioral enforcements among PSTs. For example, reducing math anxiety among PSTs (Gresham, 2012; Lake & Kelly, 2014; Vinson, 2001). Even though manipulatives have been found effective, their use in a classroom was linked with teachers' experience in using them (Moyer, 2001). PSTs beliefs and employed pedagogies were found to be linked with the mathematics method courses (Wilcox, Schram, Lappan&

Lanier, 1991; Simon & Schifter, 1991). Moreover, PSTs were found to be not equipped to use manipulatives in their instruction because of lack of training during the training years (McIntosh, 2012). It is therefore important to provide PSTs opportunity to integrate various tools that promote learning in their instruction (Quinn, 1998). None of the available research has explored efficacy of manipulatives in enhancing conceptual understanding of mathematics PSTs are required to teach at schools. The present study was an attempt to introduce manipulatives in PSTs probability classes and the findings of the present study will not only shed light on the efficacy of manipulatives in enhancing conceptual understanding of probability concepts among PSTs, but will have impact on school children's learning as the quality of teacher education can model and propel the nature of schools as learning communities.

Manipulatives in Probability Instruction

Developing conceptual understanding of probability concepts has long been a goal of mathematics instruction. Manipulatives help in developing conceptual understanding of a concept by representing a concept in many ways, and have been found to be effective in teaching probability than using symbols (Austin, 1974). Some of the manipulatives used in probability instruction include: Probability spinners, coins, playing cards, and number cubes. For example, using spinner sets can develop understanding that increasing the number of possible options would lead to a less likely event, or number cubes can be used to represent certain probability concepts (Ewbank & Ginther, 2002). The *Common Core State Standards* (CCSS, Council of Chief State School Officers [CCSSO], 2010) emphasized developing an understanding of probability by investigating chance processes and developing, using, and evaluating probability models. Discussed, next, are the two areas of instruction investigated in the present study—

conceptualizing probability and its formal computation, and theoretical and experimental probability.

Conceptualizing probability and its computation. Probability is the measure of the likelihood of an event to occur. When the events cannot be predicted with total certainty, probability is used to quantify the certainty of the events. Probability of an event is represented in terms of a number between 0 and 1 (where 0 indicates uncertainty and 1 indicates certainty). As the probability of an event increases (approximates to 1), the certainty that the event will occur increases. The use of manipulatives facilitates in understanding computations with probability by using visual representation and carrying out formal steps in computation. By using visual and tactile work, manipulatives allow conceptualizing probability and predicting an outcome prior to the actual computation, and thereby allowing justification of final computed answer. Conceptual understanding of probability enables students to understand symbolic representation without relying on the rules and evaluating whether a computation makes sense. The manipulatives allow to visualize how the sample space—available number of outcomes—affects the probability computation, and thus help in making inferences about the outcomes.

Theoretical and experimental probability. There are two approaches to compute probability—theoretical probability and experimental probability. Theoretical probability refers to the expected probability of an event whereas experimental probability is what actually happens in an event (i.e., Theoretical probability = *Number of favorable outcomes total number of outcomes* and Experimental probability = *Number of times event occurstotal number of trials*). For example, suppose we compute the probability of a coin toss. Since there are only two possible outcomes (a head or a tail), the theoretical probability of getting a head or a tail is *12.* If we compute the event of tossing the coin ten times by recording the observation and get head 3 times and tail 7 times, the experimental probability of getting a head is 310 or tail is 710. Theoretical probability is a way of estimating what could happen based on the available information; it is a calculation. Theoretical probability cannot predict what the actual results will be, but it presents likelihood of a future event. Experimental probability provides an insight to understand the actual occurrence of an event (i.e., describing event as it takes place). Manipulatives provide a useful tool in understanding the experimental probability by actually conducting an experiment, recording the observations, and comparing the results with the theoretical probability. The use of manipulatives can also facilitate in understanding why experimental probability and theoretical probability may not be the same for certain events, and how experimental probability approximates the theoretical probability as the number of trials are increased.

Research Purpose and Hypotheses

In the present study, the pre-service candidate participants received intervention (the research) in the form of instruction involving exploring the probability concepts using manipulatives. The aim of the present study was to determine if the use of manipulatives was effective in supporting PSTs' learning about key concepts in probability, in particular whether manipulatives supported students in gaining greater understanding of the formal computation of probability concepts and making inferences about certain outcome without actual computation, and understanding of the relationships between experimental and theoretical probability. In other words, the study was designed to investigate if manipulatives be used as a tool for prompting reflection and learning of probability concepts. The efficacy of the intervention was examined by investigating participants' performance in an identical pretest and posttest. Two hypotheses were tested: 1) If the intervention using manipulatives was effective, more participants will exhibit significantly greater understanding of formal computation of probability concepts and making inferences about certain outcome without actual computation; 2) If the instruction on experimental probability using manipulatives was effective, more participants will exhibit evidence of knowing and understanding experimental probability, theoretical probability, and comparison between them.

Methodology

Participants

Data were collected from students enrolled in two pre-service teacher classes enrolled in *Statistics and Probability for P-8 teachers* course. The participation was voluntary and the participants who provided the consent form were included in the study. Initially, there were 31 participants with 17 and 14 participants respectively from the two classes. One participant withdrew from the course and 5 participants were not available during the posttest. Thus, the data were collected for 25 participants with 13 and 12 participants from two classes respectively.

Data Collection Strategies

Intervention. The participants received intervention during the regular classroom hours in four sessions consisting of 1 hour and 20 minutes each. The intervention involved completing six worksheets using manipulatives. Manipulatives were used to explore concepts and included various types of number cubes, spinners, coins, and playing cards. Each worksheet had 10 questions that required participants to use a different manipulative to answer. Some of the questions in the worksheets involved large data sets (i.e., repeating an experiment 100 to 1000 times), the participants worked in groups and compiled the group data. The worksheet required participants to first carry out given computations using manipulatives, and then provide a justification and explanation of certain questions based on the findings. The questions were framed so that the participants can explain if their results were surprising to them, or how they connected with their understanding of likelihood of the event.

Probability computation. The participants completed three worksheets, individually and also in groups, using manipulatives aimed to connect concrete computation with the actual probability computation. The purpose of these worksheets was to develop participants' conceptual understanding of probability. The worksheets required participants to record outcomes of a given event using different types of manipulatives. The first activity required participants to use number cubes and spinners to explore *simple probability* (experiments completed in one step) of different outcomes. First, the participants were required to make an inference about the outcome of an event by visualizing the manipulatives, and then carrying out the computation using them. The underlying objective was to explore likelihood of an event by visually estimating the area. For example, making an inference about which event is more likely to result in rolling number 3 by looking at different sets of spinner before carrying out the computation (Figure 1). That is, probability is affected by the number of available outcomes in the sample space—lesser the total number of available options, more likely the event. In the second activity, the participants used sets of number cubes and spinners to explore *multistage probability* (experiment requiring two or more steps to complete) of compound events. For example, on two spinner sets numbered 1-6 and 1-10 making an inference and then computing the probability of getting a sum of 6 and a sum of 11, and comparing how different spinners affect the outcome (see Table 1). Also, recording and comparing the chances of getting a sum of 6 and a sum of 11 using two number cubes that are six-sided, eight-sided, and twelve-sided. In another case, tossing a coin and a six-sided number cube to understand the probability of tossing a head and rolling a 2.

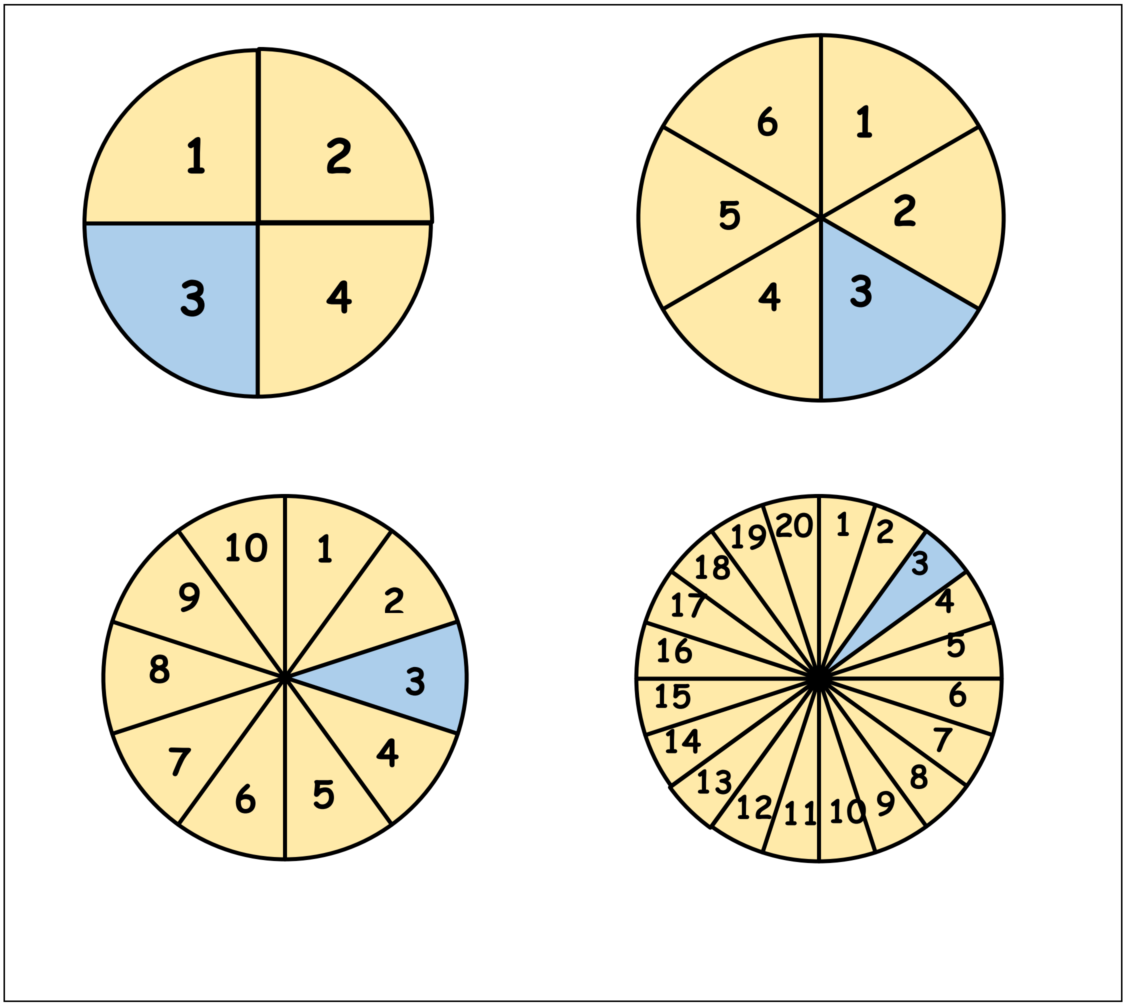

Figure 1. Comparing the area occupied by number 3 in different spinners.

These activities were included to provide hands-on experience with multistage probability computation to develop conceptual understanding. The purpose was to facilitate participants to visualize various outcomes. The third activity involved comparing different outcomes based on the changes in the sample space. For example, first, computing the probability of getting a face card (jack, queen, or king) using a standard deck of cards and then, carrying out same computation after combining two standard deck of cards. Another exploration involved tossing coin(s) for getting certain number of heads/tails. The idea was to enable understanding that the sample space (total number of outcomes) depends on number of coins used in an experiment. For example, sample space for a single coin toss is {H, T}, for two-coin toss is {HH, HT, TH, TT}, and for three-coin toss is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

The first two worksheets required the participants to use manipulatives to answer questions, and the third worksheet required them to make inferences about certain outcomes without carrying out an actual computation. The third worksheet was designed to promote logical justification of probability computations after using the manipulatives. For example, the participants were asked if we use two 1-6 sided number cubes and two 1-10 sided number cubes, which number cube is more likely to result in a sum of 7, or if tossing a coin three times resulted in 3 tails, is it more likely that the fourth toss would result a tail or a head is more likely? For developing an understanding to analyze the probability of an event and understand the meaning of various unknowns used in the computation, the participants also worked towards problems where the probability is given and the participants had to find the outcomes that could have resulted in that probability. For example, the probability of spinning a number less than 4 on a spinner numbered $1, 2, 3, \ldots, n$ is $\frac{1}{5}$; find the total number of the outcomes on the spinner? Participants were encouraged to compare the solutions in a group. This allowed them to discuss the probabilities of different outcomes and justify the solution.

Theoretical and experimental probability. The participants completed three worksheets that were aimed at developing the conceptual understanding of theoretical and experimental probability. The use of manipulatives was aimed to explore the relationship between theoretical and experimental probability, and developing an understanding of how an experimental probability (the result of an experiment) could be different from the theoretical probability (expected answer). The first worksheet included activities that required the participants to evaluate the experimental probability of given event (for example, rolling a 3 on a number cubes or tossing a coin to get a head) using fewer repetitions (10 trials), and compare it with the theoretical probability of the event. The purpose of the activity was to understand how experimental and theoretical probabilities may or may not be the same for an event. The second activity required participants to work on the same problems as in the first activity using a larger data set (100-1000 trials). Participants repeated the experiment certain number of times (or combined the data with other participants), and again compared the experimental probability with the theoretical probability of the event. For both activities, the participants were required to explain how their experimental probability (calculated results) compared with the theoretical probability (expected results). The third worksheet required participants to make inferences about certain events without carrying out a computation by sharing ideas with their peers and reflecting on other participants' computation. The aim was to foster understanding of the law of large numbers—as number of trials increase, the experimental probability approximates the theoretical probability. The participants were also required to explain the relationship between the experimental and theoretical probability, if any, based on the findings.

Testing

The participants were given an identical pre- and posttest (see Appendix A). The tests consisted of 10 multiple-choice questions (five problems based on the knowledge about probability computation and five problems based on understanding the relationship between experimental and theoretical probability), and 4 explanatory questions for examining conceptual understanding of probability. Prior to the pretest, the participants reviewed the theoretical formula for computing the probability of an outcome, and were introduced to the concept of theoretical and experimental probability. After the intervention, the participants were tested again on the identical posttest.

Results

Participants' performance on the pretest and the posttest were compared for the two concepts—conceptualizing probability and its computation, and theoretical-experimental probability using the 10 multiple-choice problems. A matched pairs t-test was used to determine if there was a significant difference between the average correct responses in the pretest and the posttest for the two concepts. For the 4 explanatory problems, participants received a score on scale of 0-4 based on appropriateness and correctness of their responses. Two testers separately scored for the explanatory problems and an average score was awarded in case of a conflict (happened less than 1% of the time). The average score was compared between the pretest and the posttest using t-test. The scores for both classes were not aggregated and kept separate to determine if there was a difference in performances between the classes.

Hypothesis 1: Conceptualizing Probability and its Computation

A paired t-test between means was conducted for the average number of correct responses in the pretest and posttest for the five probability computation multiple-choice problems. For Class 1, the mean pretest and posttest scores were 2.53 and 3.01; for Class 2, the mean pretest and posttest scores were 2.16 and 3.02, respectively. The t-statistic was marginally significant for Class 1 and Class 2 with $t(12)=1.9$, $p=.083$ and $t(11)=2.15$, $p=.053$.

A paired t-test between means for the average score in the 4 explanatory problems was significant for both classes. For class 1, the mean pretest score was 4.15 and the mean posttest score was 8.46 with $t(12)=3.22$, $p=<0.01$. For class 2, the mean pretest score was 3.92 and the mean posttest score was 7.83 with $t(12)=2.93$, $p=<0.01$.

Hypothesis 2: Theoretical and Experimental Probability

A paired t-test between means was conducted for the average number of correct responses in the pretest and the posttest for the five theoretical and experimental probability multiple-choice problems. For Class 1, the mean pretest and posttest scores were 1.69 and 2.53; for Class 2, the mean pretest and posttest scores were 1.91 and 2.91The t-statistic for Class 1 and Class 2 were significant with $t(12)=1.87$, $p=.04$ and $t(11)=2.70$, $p=.01$ respectively.

Discussion

Discussed, in turn, are the implications of the results and limitations of the present study.

Hypothesis 1: Computation of Probability

This hypothesis was made to investigate efficacy of the intervention in promoting participants' understanding of probability computations. The results for the multiple-choice problems revealing marginally significant difference and for the explanatory problems a significant difference in participants' performance between the pretest and the posttest indicated that the intervention using manipulatives was somewhat successful in imparting understanding of probability concepts and its computation among the participants. The intervention was successful to a certain extent in promoting understanding with formal computation with probability. Any positive development towards conceptual understanding of a concept is an important step. The participants understood how a formal probability computation could be affected by changes in the sample space that enabled them to logically justify an outcome without its computation. Participants' explanations for the 4 explanatory problems and the problems posed in the worksheets also revealed greater conceptual understanding of the probability computation and efficacy of the instruction using manipulatives. Instruction using manipulative enabled participants to validate/invalidate a result by justifying the computation. Two reasons could have accounted for the limited positive results of the intervention in the multiple-choice probability computation problems. First, the intervention was for a short span. Even if the intervention was successful in helping participants understand probability computation, it may take distributed practice over a long period to result in developing conceptual understanding of probability. Second, it is possible that multiple-choice problems do not present an effective outlet to investigate conceptual understanding of a concept since they do not involve providing a justification or alternate explanation of a solution.

Hypothesis 2: Theoretical and Experimental Probability

This hypothesis was made to investigate efficacy of the intervention in promoting participants' understanding of the experimental and theoretical probability. The significant difference in participants' performance between the pretest and the posttest suggested that the intervention using manipulatives was successful in promoting understanding of the experimental and theoretical probability concepts. Tactile experience with manipulatives developed participants' understanding how actual computed probability (experimental probability) may not be the same as the expected probability (theoretical probability), and the relationship between both concepts. The intervention allowed the participants to compare the results of the events between fewer trials and larger trials, which facilitated in gaining the insight how the experimental probability approximates the theoretical probability as the number of trials are increased.

Educational Implications

The results of the study suggest that manipulatives are effective tools that aid in probability instruction and promote learning through tactile work and visual representation. The use of manipulatives in probability instruction is even more beneficial as many abstract probability computations involve use of concrete objects to carry out computation and providing experiences in carrying out computations with concrete objects can foster conceptual understanding. Many participants in the present study had never dealt with playing cards and got an opportunity to use them for the first time during the intervention. Using manipulatives to do experiments and create charts or lists of outcomes enables a visual representation to interpret and analyze computations leading to gaining proficiency with probabilistic reasoning and developing a deeper understanding of sample spaces. Understanding the sample space is crucial as abundant research has shown that students of all ages do not necessarily consider sample space when carrying out probability computation (Ayres & Way, 2000; Fischbean & Schnarch, 1997). A study by Fischbein, Nello, and Marino (1991) investigated misconceptions about compound events in students' (ages 9 to 14 years) and found that majority of students did not have better understanding of sample space. For example, the tossing of two dice where the students were asked about the likelihood of two events, "a 5 on one and a 6 on another" or a "6 on both dice." Majority of the students reported that both events have the same likelihood as they did not understand that (5, 6) and (6, 5) are two different outcomes (i.e., there are two ways to get the outcome) whereas (6, 6) is one outcome as a different number appear in each number cube in the former but the same number appear on the latter.

Conclusion

The results of this intervention reveal that if chosen and used effectively, manipulatives are helpful in teaching probability and can positively affect student learning, even with college-level students. Manipulatives also provide a useful tool to prepare PSTs for teaching mathematics at schools. The theory of experimental learning is based on the idea that students gain knowledge through an active process that engages them (Hartshorn & Boren, 1990). Manipulatives facilitate in exploring mathematical ideas by stimulating curiosity, creating enjoyable learning experience, actively engaging and motivating the participants, and developing depth of understanding. The participants in the present study expressed desire to use the manipulative in their classroom for teaching probability and reported that manipulatives were engaging, useful, and informational. Manipulatives are effective instructional tools that can be used to aid mathematics instruction, but should be used with precaution and under complete guidance. As Papert (1980) calls manipulatives “objects to think with”, incorporating manipulatives can facilitate in meaningful learning of concepts with greater ease, making teaching most effective. Since students’ reflection of the instructional activities is important (Baroody, 1989), they need to be pedagogically sound that engage participants’ thinking in a meaningful way to reflect on the concepts or ideas targeted by the educator.

The key issue of effectiveness of the manipulatives in a probability instruction may not be as easily distinguished as black with white. Manipulatives are tools that students can use to model the given scenarios to make sense of the problems or the tools that can be used as objects to practice concepts. They can be used as representations by teachers to demonstrate steps and conclusions in computations. The efficacy of an intervention in promoting learning of probability concepts using manipulatives is not an all-or-nothing phenomenon. It is possible that manipulatives are beneficial in fostering understanding of certain concepts more than others. This research investigated two concepts— conceptualizing probability and its computation, and theoretical and experiment probability. A future study with other probability concepts could shed more light on the use of the manipulatives in probability instruction.

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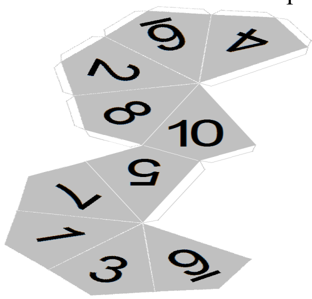
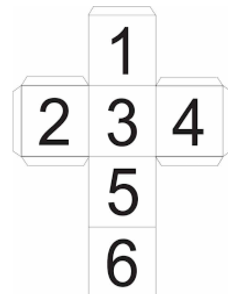
Table 1

Using a 6-sided and a 10-sided number cubes to get a sum of 6 and a sum of 11.

10-sided number cube template



6-sided number cube template



Total possible outcomes
6-sided number cube (total possible outcomes 36): Favorable outcomes for sum of 6 and sum of 11 are highlighted.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

10-sided number cubes (total possible outcomes 100): Favorable outcomes for sum of 6 and sum of 11 are highlighted.

	1	2	3	4	5	6	7	8	9	10
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	(1, 7)	(1, 8)	(1, 9)	(1, 10)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)	(2, 7)	(2, 8)	(2, 9)	(2, 10)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)	(3, 7)	(3, 8)	(3, 9)	(3, 10)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)	(4, 7)	(4, 8)	(4, 9)	(4, 10)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)	(5, 7)	(5, 8)	(5, 9)	(5, 10)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)	(6, 7)	(6, 8)	(6, 9)	(6, 10)
7	(7, 1)	(7, 2)	(7, 3)	(7, 4)	(7, 5)	(7, 6)	(7, 7)	(7, 8)	(7, 9)	(7, 10)
8	(8, 1)	(8, 2)	(8, 3)	(8, 4)	(8, 5)	(8, 6)	(8, 7)	(8, 8)	(8, 9)	(8, 10)
9	(9, 1)	(9, 2)	(9, 3)	(9, 4)	(9, 5)	(9, 6)	(9, 7)	(9, 8)	(9, 9)	(9, 10)
10	(10, 1)	(10, 2)	(10, 3)	(10, 4)	(10, 5)	(10, 6)	(10, 7)	(10, 8)	(10, 9)	(10,10)

To get a sum of 6, favorable outcomes are:
6 –sided number cube: {(5, 1), (1, 5), (4, 2), (2, 4), (3, 3)}, Probability = *536*
10-sided number cube: {(5, 1), (1, 5), (4, 2), (2, 4), (3, 3)}, Probability = *5100*
To get a sum of 11, favorable outcomes are:
6-sided number cube: {(6, 5), (5, 6)}, Probability = *236* = *118*
10-sided number cube: {(10, 1), (1, 10), (9, 2), (2, 9), (8, 3), (3, 8), (7, 4). (4, 7), (6, 5), (5, 6)}, Probability = *10100* = *110*

Appendix A

The Pretest and Posttest

All participants were given an identical pretest and posttest. The tests consisted of multiple choice (Table A1) as well as explanatory problems (Table A2).

Table A1
Circle the letter of the choice that best completes the statement or answers the question.

1. The outcomes of the theoretical probability and experimental probability for an event are
   a. Always the same
   b. Always different
   c. Depend on number of trials
   d. Cannot be predicted based on the given information
2. Suppose you want to look at chances of getting heads in a coin toss. In the first event you toss a coin 5 times and in the second event you toss the coin 100 times. It is more likely that the experimental and theoretical probabilities of
   a. Event 1 will be approximately the same
   b. Event 2 will be approximately are the same
   c. Both events will have the same experimental and theoretical probabilities
   d. Cannot be predicted based on the given information
3. A number cube with numbers 1 to 6 is rolled 600 times and chances of rolling a 6 were recorded. Which statement is more likely to be true?
   a. The cube will land on 6 exactly 100 times
   b. The cube will land on 6 approximately 600 times
   c. The cube will land on 6 approximately 100 times
   d. The cube will land on 6 exactly 200 times
4. Suppose in the first event you spin a spinner numbered 1 to 4 and in the second event a spinner numbered 1 to 20. Probability of spinning number 4 is
   a. Higher in the event 1
   b. Higher in the event 2
   c. Same for both events
   d. Cannot be predicted based on the given information
5. A coin was tossed 100 times and landings on a head or tail were recorded. It was noted that coin landed on a head 27 times and a tail 73 times (the theoretical probability of head or tail is *12* i.e., 50 times). What could be a possible reason for such discrepancy between observed and expected result?
   a. The coin is biased towards tails.
   b. The experimenter didn't toss the coin properly.
   c. The observed results may or may not be the same as the expected results. Generally, increasing the number of repetitions could lead to same results.
   d. Observed results cannot be the same as the expected results.
6. Suppose you have a 6 sided number cube. In event one, you roll a set of two number cubes with numbers 2, 2, 2, 3, 3, 3, and numbers 1, 1, 1, 5, 5, 5. In event two, you roll a set of two identical number cubes with numbers 1, 2, 3, 4, 5, 6. Probability of rolling a sum of 6 is
   a. Higher in event 1
   b. Higher in event 2
   c. Same of both events
   d. Cannot be predicted based on the given information
7. Which of the following model will result in a probability of *14?*

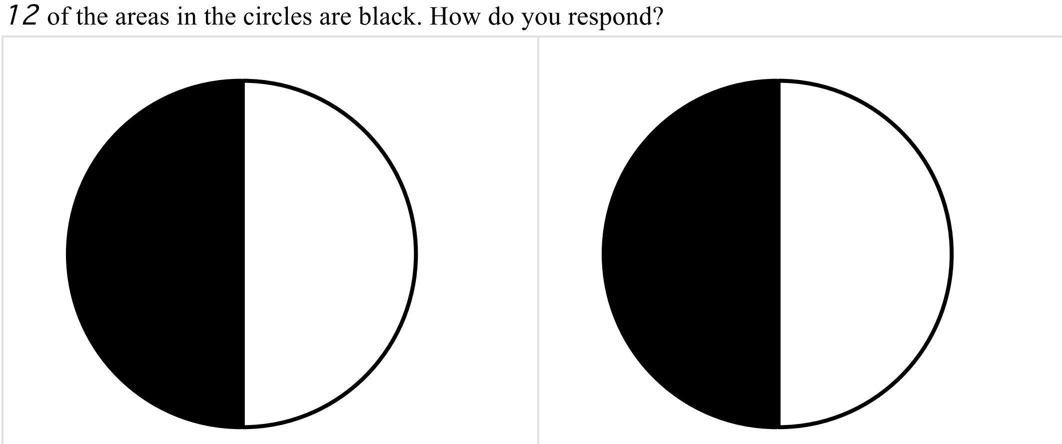
a. Rolling an even number on a number cube numbered 1-10
b. Tossing two coins to get two heads in a row
c. Spinning an even number or a prime on a spinner numbered 1-6
d. Drawing a queen from a standard deck of cards

8. Two students were predicting the outcomes of tossing a coin two times. They predicted that the chances of coin landing on tails both times as 12, chances of both coins landing on heads as 13, and chances of coin landing on a head and a tail (or a tail and a head) as 14. Is this a valid probability model?
   a. Yes, because students' prediction can be observed in a coin toss
   b. No, because students' prediction is more biased towards getting tails
   c. Cannot be predicted based on the available information
   d. No, because the total probability is not equal to 1
9. A student takes a bus to school and observed that there is a 25% chance that the bus arrives earlier than the scheduled time and 35% chance that the bus arrives after the scheduled time. If the student records the bus arrival time for 100 days, which of the following is her best prediction for the number of times the bus arrived just on scheduled time?
   a. 50
   b. 40
   c. 30
   d. 60
10. Suppose you spin a spinner numbered 1 to 10 and want to look at the chances of spinning a number between 1 to 5. The theoretical probability of spinning a number between 1 to 5 is 12. Which event is more likely to result in the experimental probability of approximately 12?
    a. Spinning the spinner 10 times
    b. Spinning the spinner 1000 times
    c. Spinning the spinner 100 times
    d. Spinning the spinner 500 times

Table A2

Provide an explanation for the problems below.

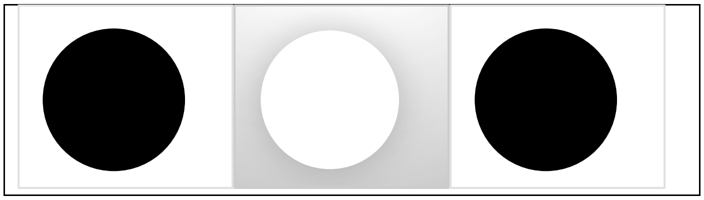
1. A student claims after looking at the spinners below that the probability of getting two blacks is 12 since 12 of the areas in the circles are black. How do you respond?



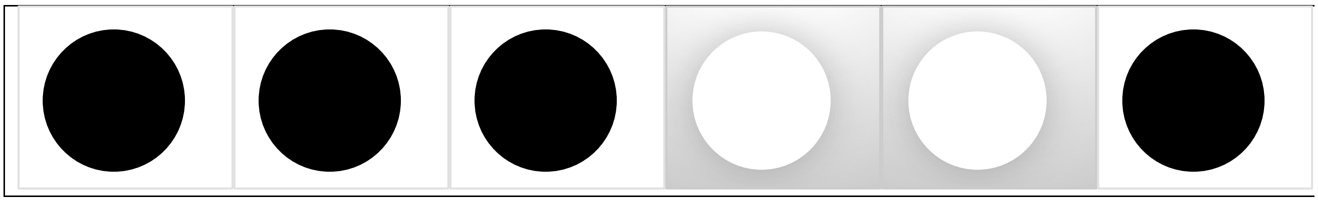
2. Referring to the two spinners above (problem 1), another student claims that the probability of getting two blacks is 13 since two blacks is one of the three possible outcomes in the sample space consisting of, two

blacks, one black and one white, two whites. How do you respond?

3. A student, looking at the two boxes below, claims that the chances of getting a white ball is higher in box 2 than box 1 since it has more white balls. How do you respond?



Box 1



Box 2

4. Referring to the two boxes above (problem 3), a student claims that in the event of drawing a ball from box 1 followed by a ball from box 2, the chances of getting two white balls is 23 since 13+26=23. Is the student right?