Using Knowledge of Story Schemas to Structure Mathematical Activity

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This study documents the implementation of mathematics curriculum organized around story schema on the classroom climate, motivation, and learning of middle school students. A theoretical framework for curricular design related to story schema is presented, and two cases of curricular design are discussed. Results revealed that the motivational appeal of story-structured curricula was much higher than traditional mathematics tasks, but the structure of the stories made generalizability of the underlying mathematics more difficult due to the specificity of students' story schema.

This paper discusses the impact of story on the nature of mathematics curricula, and the resulting difference in practice and learning that takes place as materials organized around stories are introduced into the middle school classroom. It is an attempt to document the fundamental differences in philosophy, practice, and outcomes (for lack of a better word) that a mathematics program based on the National Council of Teachers of Mathematics' (NCTM) *Standards* documents (1989, 1991) can demonstrate in the real world with real teachers and real students. First, a discussion of how stories serve as an aid to reduce cognitive load and to augment cognitive organization of mathematical concepts will be presented. Later, two case studies describing the implementation of two units that are built around a story will be described.

In the not-so-recent past, human cultures transmitted their values, knowledge, and rituals via oral tradition. Storytellers held a revered place in society, and their works inform cultures about our archetypal values. In the oral traditions of indigenous cultures, the knowledge and customs of the society are still transmitted to children through folklore-stories

The difference between indigenous cultures and Western society in the use of the story as a medium for learning is apparent when one looks at the age of the learner in which the story ceases to be used as an important teaching tool. In indigenous cultures, stories are related, and are participated in, by each member of the society. Adults engage in story listening as well as storytelling as a way of rediscovering what is basic and important to their culture.

In Western culture, the story has been removed from the teaching/learning interaction usually by the elementary or middle school years, and often sooner. In the case of mathematics, we attempt to create "story problems," to situate the content domain of mathematics within real-life applications, but these "stories" have neither the structure, function, or appeal associated with a true story.

In the whole language literature, the argument has been put forth that children should be exposed to reading literature with rich vocabulary and linguistic patterns, to illustrate grammatical rules, composition structure and style. When many literary examples are explored, students ultimately find models for personal expression (e.g., Ralston, 1991). In mathematics education, the same goals also apply. Students who are exposed to mathematical stories can discover the underlying assumptions behind different modes of mathematical inquiry, uncover and invent rules, structure and concepts, and, ultimately construct mathematical models for personal expression and utility.

What characteristics of stories can be applied to modem mathematics education to make instruction more meaningful? First, stories tend to have a definite structure that is easily learned. They provide a definite introduction, some plot that situates the character and conflict into an event history, and a resolution that brings the listener (reader) to a higher level of understanding, be it of his/her culture, of the self, or of the world. This structure is so highly adaptable that nearly every story an individual encounters in his or her life will fit nicely into it.

Second, a story in its most classical sense has a *theme.* The theme may be philosophical (dealing with underlying meaning), archetypal (e.g., dealing with elemental experiences), or explanatory (dealing with how something works), but usually is a combination of two or more of these. The notion of theme embodies a sense of continuity, a feeling of purpose, and a unification of important ideas. As in the literary sense, themes in mathematics consist of organizing ideas that link and direct the cognitive structuring of mathematical concepts. The theme of *ratio* for example, connects important concepts in geometry (similarity), algebra (linearity), number (scaling), and calculus (differentiation).

Third, and most important, in the learning situation, the story gives direction to the students' work and provides avenues for reflection to consider a situation more carefully. It requires the development of concepts and strategies on the part of the reader instead of offering them as givens (the analogy of a mystery fits here nicely). Students can see the progression of their knowledge from the basic level to more sophisticated thinking by "retelling" the story, using new information.

Lastly, stories are appealing. Such a connection to the students' personal world in a mathematics setting can make the mathematics real for the student-it tells her something more about things that are of interest or of importance to her.

Stories are powerful teaching tools because they are processed cognitively in a special way-through "story schema."

Story Schema

Mandler (1984) describes "story schema" as cognitive abstractions of the *structure* of literary expressions we read or hear. People establish higher-order representations that dictate the necessary rules for event sequencing, and that suggest other structures in memory that can be used to "fill in the gaps" when necessary knowledge is omitted from the text. For example, the simple statement, *Jill put on her hiking boots, got into her car and drove off to the mountains* is descriptive enough to tell us that the character is a woman, drives a car, and that she wears boots. However, we are left to fill in the necessary action that makes Jill's trip possible. We imply that if she put on her hiking boots, she must tie up the laces. If she drives her car, she must insert the key, turn on the engine, step on the accelerator, etc. Our story schema in essence calls up familiar representations (i.e., *"tying your shoes")* that make explicitly stating all of the necessary information unnecessary.

The benefits of the story schema echo those of the script representation (Abelson, 1981; Schank & Abelson, 1977). Namely, they serve three important functions as cognitive structures: 1) They place stereotyped events into readily retrievable representations; 2) They are highly organized-both temporally and conceptually; and 3) because they are habitualized, cognitive load is "freed up" for more efficient use of working memory.

Romberg and Tufte (1987) argue that the organization of stories are such that events are related in a simultaneously causal and hierarchical fashion-that is to say, certain events initiate other events, and subsequent events necessarily follow their antecedents and suggest future actions. The story schema organizes these structural aspects of stories into a plausible chain-of-event possibilities early in the process of reading or listening to the tale. As new information is encountered, one's schema assimilates the new data and creates space for additional information by eliminating implausible event possibilities and attaching possibilities more congruent with the new information. As the connections between the structure and theme of the story are made, the story becomes a self-perpetuating system. As new information is added, the reader does not need to develop a new representation of the plot; he or she merely needs to add the new information and access relatively few new ideas to draw it together.. The key notion here is that in this situation, unlike "story problems," students do not have to make sense of an entirely new context when a new exercise is presented. Rather, they can use the information they have already learned, discover the new mathematical relationship being developed, and apply it to the already-accessed story schema for later retrieval..

As the story schema applies well to the benefits of using stories as contexts for situating mathematical content, it also applies well to the drawbacks-namely lack of generalizability. The knowledge a student develops working through a story may be isolated to the context of the story or closely related situations. Thus, some opportunity for generalization must be built into the story through sub-topics or "digressions" in the body of the unit, although how much digression is tolerable is a subject for debate and study.

The project described in this study has developed and pilot-tested several units based on these principles, and each tells a story that compels students to delve further into mathematical inquiry. The remainder of this paper is devoted to a brief description of two of these units and the results of their implementation in middle school classrooms.

Method

Participants

Seventeen middle school teachers and their students (338 5th grade, 61 7th grade, and 110 8th grade students) from four Midwestern school districts participated in the study. Each teacher had at least one other teacher in his or her school who was also participating in the project.. This pairing allowed teachers to collaborate on lesson planning, troubleshooting, and team teaching if appropriate.

Procedure

Prior to teaching any units, teachers were involved in two half-day inservice sessions. As the teachers began teaching their initial units, two additional half-day inservice meetings were scheduled to discuss problems of implementation and assessment Issues.

The primary participant observer of each unit (i.e., the person with the most familiarity with the mathematical content and the anticipated pedagogical issues) observed classrooms at least twice per week for the four to five weeks of the pilot test. Additional staff observed classes periodically to "fill-in" if the primary adaptor was absent, and to provide a second opinion as to what was happening in the classes. During the observations, researchers would question students about their strategies and their affect, and would solicit opinions about how to improve the unit. During the teaching of each unit, teachers were expected to annotate their version of the teachers' guide, describing their adaptations of the units, correcting errors, and describing students' strategies for solving problems. In addition, to assist teachers in productive reflection over their practice, we encouraged teachers to keep a journal of their experiences, thoughts, and frustrations.

Following the completion of each unit, pilot teachers were interviewed individually. These interviews were scheduled to follow the observed lessons as closely as possible. Interview questions focused on the goals of the unit, the characteristics of the units and teachers' guides that facilitated teaching and learning, and the characteristics of the units that hindered students' learning.

During the interview, the researcher and teacher worked through the unit, highlighting areas where students and teachers had difficulties. Teachers were asked to describe the nature of the difficulty, how they attempted to remedy the problem, and the strategies students used to solve the problem.

Mathematics, Measurement, and Me

The Unit

In this unit (Clarke, in press), students are given the task of solving the "mystery bone" problem. A human bone has been found and students have to piece together information about the mystery person, using the data that they collect about themselves

and their class members. They explore the relationships between the sizes of various parts of the body and discover the importance of surface area in the field of medicine. The unit links mathematics to forensic science, art, and health.

The unit begins with the teacher referring the students to the newspaper headline shown below:

Bone Puzzle Baffles Detectives

> Detectives have found a bone buried in a hole on the banks of Lake Kegonsa in Stoughton. Forensic anthropologists have established that the bone is a radius bone, and that it certainly belongs to a human being. The bone was 28 cm long. Police don't know the gender of the person yet. The detectives hope soon to provide a clearer picture of what this person may have looked like. Investigations are continuing.

The students are then asked to provide further information about the "mystery person" to whom the bone belongs. They are encouraged to take a variety of measurements (e.g., height, radius bone) of students in their small group, in their search for patterns to construct a better picture of the mystery person. Following the activity, the groups are asked to outline their findings, clearly explaining their conclusions, as the teacher attempts to "referee" the discussion. After several activities of this kind, the students are expected to write a forensic report, outlining their results, and the process that led to them.

Other activities within the unit provides the opportunity for students to explore relationships between various parts of their body. As more information is given, the students gain a better picture of what the missing person looks like, and use the information to solve the mystery at the end of the unit.

Observed Classes

Students were not used to studying contextualized mathematics and had particular difficulty reading several paragraphs before solving a mathematics problem. The difficulty did not lie, in general, in the reading level of the students. Rather, the concept of *mathematizing*-~~dr~~*awing* the mathematical relationships out of a realistic and, hence, complex situation-was foreign to the students. When they were having difficulty, teachers sometimes read certain of the longer passages with the whole class, assessing whether the students had grasped the problem setting, and then let the students work on the problems.

Teachers found that it was important to "pull all the thoughts together" at various times during the unit. They often went back to the beginning of the story of the mystery bone and retold the tale, highlighting the crucial information.

The observations of the unit made it obvious that most students had had almost no previous experience measuring anything, whether in metric or customary units, and as a result, the development of measurement skills became a major goal for the teachers.

The overall conclusion about the unit was that it was enjoyed by the students, with students who normally had great difficulty with the subject producing excellent work; but the teaching approaches involved in the early sections were very challenging for most teachers. Teachers commented on students becoming more independent learners as the unit progressed, and more comfortable working in small groups. Teachers noted that students learned more about measurement and the metric system, the use of tabular information to display data, the use of formula-to estimate and predict from data, and gained more practice in basic arithmetic skills in the context of solving real problems than in more routine mathematics classes using their traditional text. One teacher commented:

> At this point, 21 out of 22 students would like to spend the whole year at this kind of math. Only 1 prefers the textbook-its predictable and she's good at it. Several said they enjoy measuring each other and discussing their answers.

Another wrote in her journal:

> My overall impressions are positive, especially related to the active, engaged learning I observe in students (although the noise level often gets too loud!).

One of the most important comments that both teachers and students made about the unit, and the one that is the most telling with respect to the motivational power of the story context, was that they felt that the theme of the "mystery bone" got lost in the latter parts of the unit. Students found the context motivating and unifying, but were very disappointed when they could not follow the mystery through to the end of the unit to determine exactly who the deceased was. Teachers identified the continuity of this theme as important. The story and the need to solve the crime became a highly motivating force in the unit --it provided interest and relevance, and it was important that this was maintained throughout the unit. As a result, the "mystery person" became the central theme in a revised version and is used in a final assessment to draw all of the information in the story together in a satisfactory forensic report.

When students were put in the position of "detective," their creative interpretations of the mathematics became apparent. When the students in one class were given two bone measurements-radius bone of 35 cm, and a tibia bone of 46 cm-and were asked, "Do *you* believe that the two bones belonged to the same person? Explain your reasoning," one student replied, "I think they belong to two different people because the predicted height based on the radius measurement is about [3.3 x (radius) + 81 cm] 196 cm, and the predicted height based on the tibia measurement is [2.4 x (tibia) + 75 em] 185 em. That's a big difference."

His partner disagreed: "Come on. What do you think the probability is that someone would dump two stiffs into the same hole?"

Another student said, "I think the 'person' was really a gorilla." When asked why he thought so, he replied, "Because the two bones are way out of proportion. The radius is too long, and the tibia is too short, so it looks more like a gorilla than a human." Argument ensued. The teacher used this discrepancy to discuss with the class the notion of errors in prediction. This discussion illustrates the fact that numbers get their meaning from the reality of the student, and do not exist independent of context. The story structure allowed students to use *their* reality-i.e., their understandings of proportions and their informal knowledge-to solve the mystery.

Decision Making

The Unit

In this unit for Grade 7 (Roodhardt, Middleton, & Burrill, in press), students are presented with a dilemma: Two parties with competing points of view are negotiating on rezoning a reclaimed landfill to build new residences for the city center. One party wants to maximize the number of people who can live in the area, and so wants to build high-rise apartment houses. The other party wants to rezone the land to accommodate the need for new home buyers, and therefore wants single-family houses. As the two parties negotiate, students are asked to develop feasible plans for making decisions on just how the land will be zoned and to make recommendations as to how these plans can be carried out. The major mathematical content of the unit centers on the graphical representation of linear inequalities-the basis for linear programming. However, the abstraction of the graphical information into systems of algebraic inequalities was not emphasized in the unit. Rather, the concepts of "fair exchange" and "trade-off' were used to help students understand the relationship between x and y values.

Students were encouraged to take the position of one of the parties and argue their position with their classmates. By the end of the unit, students develop a plan for rezoning the land given certain city regulations.

<u>Observed Classes</u>

The most surprising feature of the observations for *Decision Making* is that students in all of the courses could make sense of the context and understand the nature of the mathematics involved at approximately the same level[1]. The 8th-grade students did progress more rapidly through the unit than the 7th-grade, as did the algebra and accelerated classes, but the nature of the knowledge and the sophistication of solution strategies were similar in all classes. Students took to the new structure and style of mathematics readily. It was clear that the context of the story, rezoning an area of land situated between an inner-city area and a suburban area, was familiar to the students. At first, there was some tendency to try to go too quickly through the exercises without considering the story. However, once students returned to the story context (coming up with a plan that satisfied both parties), they were better able to reason effectively about the situations given.

It became apparent as students progressed through the unit that the sense of story-the plot-carried their thinking to deeper levels than the static presentation of mathematical concepts. This was especially clear during the first assessment when the teacher gave a short exposition on the notion of slope, and then assigned a test that provided a set of linear ordered pairs and asked the students to find the "slope" of the line. Students were unable to grasp the notion of slope in the abstract.. However, when the terms "fair exchange" and "trade-off' were used, and the problems related to the plot of the story, students replied, "Oh, yeah, it is 2 Low buildings for 3 High buildings," for a slope of 3/2.

When students were questioned with regard to their strategies, they often referred back to the story and said, "Oh, its just like..." and used their previous strategies as analogies to communicate their present thinking. When the observers asked the students what the mathematical constraints to the systems of inequalities meant, the students used the relationships in the story to come up with a general solution.

At the end of the unit, all of the teachers elected to have the students create a scale drawing of a proposal they felt was an acceptable, equitable solution to the problem of rezoning the land in the story. Students felt that merely designing the proposed subdivision was not visual enough, so they created models of their housing and placed them in a map of the available area to justify their solutions. Student projects were highly creative and displayed a diversity of opinions. For four of the six classes, students were not allowed class time to work on their projects. Students met after school and on the weekends to develop the scale models of the subdivision. No two projects were alike, and all but two projects went beyond the requirements for the assessment put forth by the teacher..

The affect of the students while engaged in the unit was generally very high and very few of the students in both classes did poorly-and most did better than usual on the assessment activities. The end-of-unit projects displayed mastery of the basic concepts of the graphical representation of the data, the use of linear systems to explore the feasibility of different plans in the housing context, the application of the mathematics to difficult real world problems with multiple solutions possible, and the extraordinary motivation of students in the process of learning interesting and important mathematics.

At the end of the unit, students were asked what they thought about it. Most agreed that it was hard, but not too hard to do. They felt it was interesting because it dealt with real-life issues that they had heard about before. They understood the application of the context to the fields of architecture and urban design, and several students expressed a desire to learn more about these professions.

When the students in two classes were asked what they considered the worst part of the unit, they alluded to a section in the middle of the unit that departs from the story theme. Originally, this section was designed to provide more avenues for generalizability of the mathematics to diverse contexts, such as the trade-off between relative humidity and temperature in creating a safe environment for strenuous athletic activities. The students felt that this departure was unnecessary and detracted from the flow of the unit. They lost interest in this section of the unit, and some indicated that they became irritated because it kept them from pursuing the story of the housing battle. They suggested that we revise the unit and omit this section.

Although the unit was difficult, the students expressed a desire for more mathematics of this type, and concomitantly, a desire **not** to return to the regular text.

The teachers were all very positive about the unit; in particular, they cited the ability of the story-line to provide an organizing theme for the instruction. They all indicated that when students had difficulty, they would just go back and think of the story to put the problem into perspective and find a way to a solution. They did lament that because the unit was organized around a story, it was extremely difficult for students to catch up with the class if they had been absent for a couple of days. The absent students did not know the new twists in the plot that had developed while they were gone and exhibited confusion when they tried to enter into group discussion. This difficulty was echoed by the students.

Discussion

The results of this study clearly illustrate both the advantages and disadvantages of structuring mathematics curricula around story contexts. Students in the classrooms studied learned a great deal of complicated mathematics. However, because the story format brings in a large number of mathematical ideas as characters, assessment of students' mathematical knowledge does not fit neatly into a testing structure that focuses on isolated topics. Rather, the assessment strategies that most accurately reflected what students learned as a result of the units mirrored the story structure of the units. In *Mathematics, Measurement, and Me,* students demonstrated their understanding by compiling a forensic report that showed how knowledge of measurement, simple algebra, and data analysis were combined to solve the mystery of the missing person. In *Decision Making,* students combined all of the information gathered during instruction and built scale models of a subdivision that represented a fair compromise between two opposing viewpoints. The models required understanding of graphing, linear inequalities, and algebraic manipulation of data. In both cases, assessment required the communication of mathematical ideas and perspective taking.

The two units also proved to be an equalizing factor in the classrooms studied. Teachers reported seeing students who normally have trouble with mathematics (presented in the traditional, isolated manner) blossoming as they saw connections to the real world. Students found that they could relate to the stories; they felt that the story contexts were important applications of mathematics, that they were interesting situations to study, and that the mathematics became more meaningful in a personal sense for them. Moreover, the students who were normally high achievers did not achieve at a lower level, but they did express some initial discomfort and difficulty with the new units. This may be due to the conceptual nature of the stories. Traditional algorithms, in fact, often hindered finding appropriate solutions to the problems presented. Students comfortable with such procedures may have experienced more difficulty than they had previously encountered in mathematics class and were forced to rethink their conceptions of mathematics.

Although struggling students tended to do well in these units, the highly interconnected nature of the stories made it extremely difficult for absent students to catch up. This situation is analogous to the person who gets up to buy popcorn in the middle of a movie. When they return, the plot may have changed dramatically. This not only confused the absent students, but also irritated the students who had to back up and retell the story.

Perhaps the most compelling feature that we found in observing the units was the motivational impact they had on the students. Students in all the classes studied reported that they enjoyed the new mathematics and made it clear that they would prefer this type of curriculum over their textbooks. Although the novelty of the situation certainly added to their enjoyment, the sustained effort students demonstrated over four to six weeks of work would suggest that the unit structure and content tapped into the structure of students' intrinsic motivations (e.g., Middleton, 1993). The assessment activities in the *Decision Making* unit in particular attest not only to a high degree of conceptual understanding, but also demonstrated students' willingness to go beyond expectations for the sake of creating a quality product.. Unmotivated students just do not work after school and on the weekends to create high quality, mathematically correct scale models merely for the sake of obtaining a good grade.

For teachers, the unit was refreshing and exciting. It allowed them to find local situations that fit into the mathematics and the context of the units and to use their own creativity to structure a productive learning environment.. However, teachers did indicate that this type of curriculum is more difficult to teach, monitor, and assess. One teacher lamented, "I **know** these kids have learned more in the past three weeks than they have in the whole semester (the prior six weeks), but I don't know how to assess it."

This study did not address one major concern: that structuring mathematics curricula into story contexts could lead to a lack of generalizability of the underlying mathematics due to the specificity of the story schema. We found that providing diversions even caused frustration in some of the students being observed. Future research on the power of stories in mathematics education should address this issue.

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